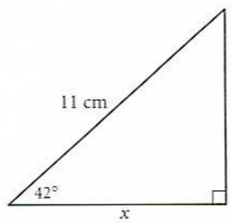
You can use trigonometric ratios to find unknown side lengths of a right triangle given the measures of any side and any acute angle. Read Example B in your book and then read Example A below.

EXAMPLE A Find the value of x.



Solution You need to find the length of the leg adjacent to the 42° angle. You are given the length of the hypotenuse. The trigonometric ratio that relates the adjacent leg and the hypotenuse is the cosine ratio.

$$\cos 42° = \frac{x}{11}$$

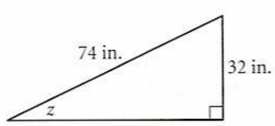
$$11 \cdot \cos 42° = x$$ Multiply both sides by 11.

$$8.17 \approx x$$ Use your calculator to find cos 42° and multiply the result by 11.

The value of x is about 8.2 cm.

If you know the lengths of any two sides of a right triangle, you can use *inverse trigonometric functions* to find the angle measures. Example C in your book shows how to use the inverse tangent, or $\tan^{-1}$, function. The example below uses the inverse sine, or $\sin^{-1}$, function.

EXAMPLE B Find the measure of the angle opposite the 32-inch leg.



Solution You are given the lengths of the leg opposite the angle and the hypotenuse. The ratio that relates these lengths is the sine ratio.

$$\sin z = \frac{32}{74}$$

$$\sin^{-1}(\sin z) = \sin^{-1}\left(\frac{32}{74}\right)$$ Take the inverse sine of both sides.

$$z = \sin^{-1}\left(\frac{32}{74}\right)$$ The inverse sine function undoes the sine function.

$$z \approx 25.6°$$ Use your calculator to find $\sin^{-1}\left(\frac{32}{74}\right)$.

The measure of the angle opposite the 32-inch side is about 26°.

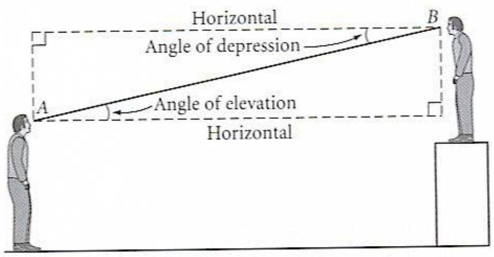
CONDENSED LESSON

12.2 Problem Solving with Right Triangles

In this lesson you will

- Use trigonometry to **solve problems involving right triangles**

Right triangle trigonometry is often used to find the height of a tall object indirectly. To solve a problem of this type, measure the angle from the horizontal to your line of sight when you look at the top or bottom of the object.

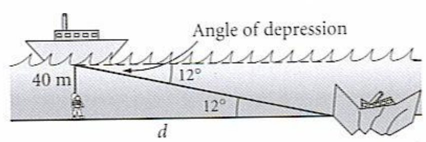


If you look up, you measure the **angle of elevation.** If you look down, you measure the **angle of depression.**

The example in your book uses the angle of elevation to find a distance indirectly. Read the example carefully. Try to solve the problem on your own before reading the solution. Then try to solve the problems in the examples below. Example A is Exercise 13 in your book. It involves an angle of depression.

EXAMPLE A A salvage ship's sonar locates wreckage at a 12° angle of depression. A diver is lowered 40 meters to the ocean floor. How far does the diver need to walk along the ocean floor to the wreckage?

Solution Make a sketch to illustrate the situation. Notice that because the ocean floor is parallel to the surface of the water, the angle of elevation from the wreckage to the ship is equal to the angle of depression from the ship to the wreckage (by the AIA Conjecture).



The distance the diver is lowered (40 m) is the length of the leg opposite the 12° angle. The distance the diver must walk is the length of the leg adjacent to the 12° angle. Set up the tangent ratio.

$$\tan 12° = \frac{40}{d}$$

$$d\tan 12° = 40$$

$$d = \frac{40}{\tan 12°}$$

$$d \approx 188.19$$

The diver must walk approximately 188 meters to reach the wreckage.

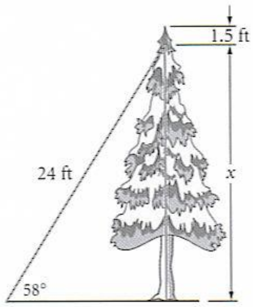
(continued)

EXAMPLE B

An evergreen tree is supported by a wire extending from 1.5 feet below the top of the tree to a stake in the ground. The wire is 24 feet long and forms a 58° angle with the ground. How tall is the tree?

▶ ***Solution***

Make a sketch to illustrate the situation.



The length of the hypotenuse is given, and the unknown distance is the length of the side opposite the 58° angle. Set up the sine ratio.

$$\sin 58° = \frac{x}{24}$$

$$24 \cdot \sin 58° = x$$

$$20.4 \approx x$$

The distance from the ground to the point where the wire is attached to the tree is about 20.4 feet. Because the wire is attached 1.5 feet from the top of the tree, the tree's height is about 20.4 + 1.5, or 21.9 feet.

CONDENSED LESSON

12.3 The Law of Sines

In this lesson you will

- Find the **area of a triangle** when you know two side lengths and the measure of the included angle
- Derive the **Law of Sines,** which relates the side lengths of a triangle to the sines of the angle measures
- **Use the Law of Sines to find an unknown side length** of a triangle when you know the measures of two angles and one side **or to find an unknown *acute* angle measure** when you know the measures of two sides and one angle

You have used trigonometry to solve problems involving right triangles. In the next two lessons you will see that you can use trigonometry with *any* triangle.

Example A in your book gives the lengths of two sides of a triangle and the measure of the included angle and shows you how to find the area. Read the example carefully. In the next investigation you will generalize the method used in the example.

Investigation 1: Area of a Triangle

Step 1 gives three triangles with the lengths of two sides and the measure of the included angle labeled. Use Example A as a guide to find the area of each triangle. Here is a solution to part b.

b. First find h.

$$\sin 72° = \frac{h}{21}$$

$$21 \cdot \sin 72° = h$$

Now find the area.

$$A = 0.5bh$$

$$A = 0.5(38.5)(21 \cdot \sin 72°)$$

$$A \approx 384.46$$

The area is about 384 cm^2.

Then use the triangle shown in Step 2 to derive a general formula. The conjecture below summarizes the results.

SAS Triangle Area Conjecture The area of a triangle is given by the formula $A = \frac{1}{2}ab \sin C$, where a and b are the lengths of two sides and C is the angle between them. **C-100**

(continued)

You can use what you've learned to derive the property called the Law of Sines.

Investigation 2: The Law of Sines

Complete Steps 1–3 in your book. Below are the results you should find.

Step 1 $\sin B = \frac{h}{a}$, so $h = a \sin B$

Step 2 $\sin A = \frac{h}{b}$, so $h = b \sin A$

Step 3 Because both $b \sin A$ and $a \sin B$ are equal to h, you can set them equal to one another.

$$b \sin A = a \sin B$$

$$\frac{b \sin A}{ab} = \frac{a \sin B}{ab} \quad \text{Divide both sides by } ab.$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{Simplify.}$$

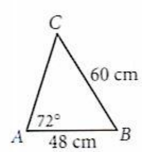
Now complete Steps 4–6. Combine Steps 3 and 6 to get this conjecture.

Law of Sines For a triangle with angles A, B, and C and sides of lengths a, b, and c (a opposite A, b opposite B, and c opposite C), $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. **C-101**

Example B in your book shows you how to use the Law of Sines to find the lengths of a triangle's sides when you know one side length and two angle measures. Try to solve the problem yourself before reading the solution.

Read the text before Example C, which explains that you can use the Law of Sines to find the measure of a missing angle *only if* you know whether the angle is acute or obtuse. You will only be asked to find acute angle measures. Example C shows you how to do this. Here is another example.

EXAMPLE Find the measure of acute angle C.



▶ ***Solution*** Use the Law of Sines.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\sin C = \frac{c \sin A}{a}$$

$$\sin C = \frac{48 \sin 72°}{60}$$

$$C = \sin^{-1}\left(\frac{48 \sin 72°}{60}\right)$$

$$C \approx 49.54$$

The measure of $\angle C$ is approximately 50°.

CONDENSED LESSON

12.4 The Law of Cosines

In this lesson you will

- Use the **Law of Cosines** to find side lengths and angle measures in a triangle

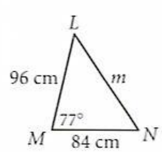
You have solved many problems by using the Pythagorean Theorem. The Pythagorean Theorem is a very powerful problem-solving tool, but it is limited to right triangles. There is a more general relationship that applies to all triangles.

Think of a right angle that is made by a hinge with two legs of fixed length as its sides. What happens to the length of the third side (the hypotenuse when the angle measures 90°) and to the Pythagorean relationship as the hinge closes to less than a right angle or opens to more than a right angle? To explore this question, look at the triangles pictured at the top of page 661 and read the paragraphs that follow, including the Law of Cosines. Add the Law of Cosines to your conjecture list.

The Law of Cosines works for both acute and obtuse triangles. In your book, read the derivation of the Law of Cosines for acute triangles on page 662. In Example A, the Law of Cosines is used to find the length of the third side of a triangle when you are given the lengths of two sides and the measure of their included angle. Read Example A in your book. Then work through Example A below.

EXAMPLE A

Find m, the length of side $\overline{NL}$ in acute $\triangle LMN$.



Solution

Use the Law of Cosines and solve for m.

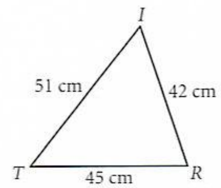
$c^2 = a^2 + b^2 - 2ab\cos C$	The Law of Cosines.
$m^2 = 96^2 + 84^2 - 2(96)(84)(\cos 77°)$	Substitute m for c, 96 for a, 84 for b, and 77° for C.
$m = \sqrt{96^2 + 84^2 - 2(96)(84)(\cos 77°)}$	Take the positive square root of both sides.
$m \approx 112.45$	Evaluate.

The length of side $\overline{NL}$ is about 112 cm.

(continued)

Example B in your book uses the Law of Cosines to find an angle measure. Here is another example. Solve the problem yourself before reading the solution.

EXAMPLE B Find the measure of $\angle I$ in $\triangle TRI$.



▶ ***Solution*** Use the Law of Cosines and solve for I.

$$c^2 = a^2 + b^2 - 2ab\cos C$$ The Law of Cosines.

$$45^2 = 51^2 + 42^2 - 2(51)(42)(\cos I)$$ Substitute 45 for c, 51 for a, 42 for b, and I for C.

$$\cos I = \frac{45^2 - 51^2 - 42^2}{-2(51)(42)}$$ Solve for $\cos I$.

$$I = \cos^{-1}\left(\frac{45^2 - 51^2 - 42^2}{-2(51)(42)}\right)$$ Take the inverse cosine of both sides.

$$I \approx 56.89$$ Evaluate.

The measure of $\angle I$ is about 57°.

CONDENSED LESSON

12.5 Problem Solving with Trigonometry

In this lesson you will

- Use trigonometry to solve problems, including problems that involve **vectors**

Some of the practical applications of trigonometry involve vectors. In earlier vector activities, you used a ruler and a protractor to measure the size of the resulting vector and the angle between vectors. Now you will be able to calculate resulting vectors by using the Law of Sines and the Law of Cosines.

In the example in your book, the Law of Cosines is used to find the length of a resultant vector and the Law of Sines is used to find its direction. Read the example and make sure you understand each step.

The example below is Exercise 5 in your book. Try to solve the problem on your own before reading the solution.

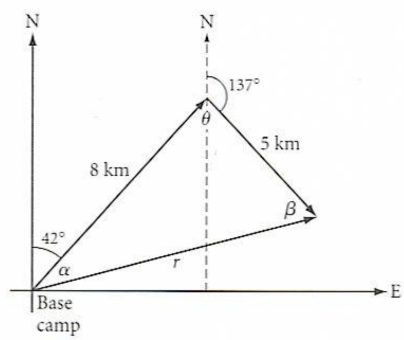
EXAMPLE Annie and Sashi are backpacking in the Sierra Nevada. They walk 8 km from their base camp at a bearing of 42°. After lunch, they change direction to a bearing of 137° and walk another 5 km.

a. How far are Annie and Sashi from their base camp?

b. At what bearing must Sashi and Annie travel to return to their base camp?

▸ Solution

a. Draw a diagram to illustrate the situation. (Remember, a bearing is measured clockwise from north.) Here, the distance from base camp is r. To find r, you can find the value of θ and then use the Law of Cosines.



Think of θ as being made up of two parts, the part to the left of the vertical and the part to the right. Using the AIA Conjecture, the part to the left has measure 42°. Because the part to the right and the 137° angle are a linear pair, the part to the right has measure 43°. So, the measure of θ is 42° + 43°, or 85°. Now use the Law of Cosines.

$$r^2 = 8^2 + 5^2 - 2(8)(5)(\cos 85°)$$

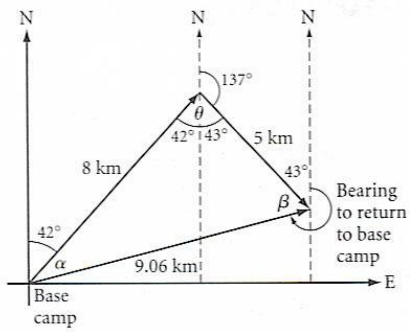
$$r = \sqrt{8^2 + 5^2 - 2(8)(5)(\cos 85°)}$$

$$r \approx 9.06$$

Sashi and Annie are about 9.1 km from their base camp.

(continued)

b. Add the information you found in part a to the diagram.



The diagram indicates that the bearing Sashi and Annie must travel to return to the base camp is $360° - (43° + \beta)$. To find β, use the Law of Sines.

$$\frac{\sin\beta}{8} \approx \frac{\sin 85°}{9.06}$$

$$\sin\beta \approx \frac{8\sin 85°}{9.06}$$

$$\beta \approx \sin^{-1}\left(\frac{8\sin 85°}{9.06}\right)$$

$$\beta \approx 61.6$$

β is about 62°, so the bearing is about $360° - (43° + 62°)$, or 255°.

CONDENSED LESSON

12.1 Trigonometric Ratios

In this lesson you will

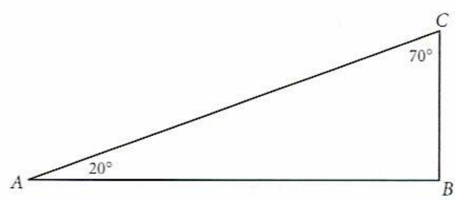
- Learn about the trigonometric ratios **sine, cosine,** and **tangent**
- Use trigonometric ratios to **find unknown side lengths** in right triangles
- Use **inverse trigonometric functions** to **find unknown angle measures** in right triangles

Read up to Example A in your book. Your book explains that, in any right triangle with an acute angle of a given measure, the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle is the same. The ratio is known as the **tangent** of the angle. Example A uses the fact that $\tan 31° \approx \frac{3}{5}$ to solve a problem. Read the example carefully.

In addition to tangent, mathematicians have named five other ratios relating the side lengths of right triangles. In this book, you will work with three ratios: **sine, cosine,** and **tangent,** abbreviated sin, cos, and tan. These ratios are defined on pages 641–642 of your book.

Investigation: Trigonometric Tables

Measure the side lengths of $\triangle ABC$ to the nearest millimeter. Then use the side lengths and the definitions of sine, cosine, and tangent to fill in the "First $\triangle$" row of the table. Express the ratios as decimals to the nearest thousandth.



	$m\angle A$	$\sin A$	$\cos A$	$\tan A$	$m\angle C$	$\sin C$	$\cos C$	$\tan C$
First $\triangle$	20°				70°			
Second $\triangle$	20°				70°			
Average	—				—			

Now use your protractor to draw a different right triangle ABC, with $m\angle A = 20°$ and $m\angle C = 70°$. Measure the sides to the nearest millimeter and fill in the "Second $\triangle$" row of the table.

Calculate the average for each ratio and put the results in the last row of the table. Look for patterns in your table. You should find that $\sin 20° = \cos 70°$ and $\sin 70° = \cos 20°$. Notice also that $\tan 20° = \frac{1}{\tan 70°}$ and $\tan 70° = \frac{1}{\tan 20°}$. Use the definitions of sine, cosine, and tangent to explain why these relationships exist.

You can use your calculator to find the sine, cosine, or tangent of any angle. Experiment with your calculator until you figure out how to do this. Then use your calculator to find $\sin 20°$, $\cos 20°$, $\tan 20°$, $\sin 70°$, $\cos 70°$, and $\tan 70°$. Compare the results to the ratios you found by measuring sides.

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